DISCRETE STRUCTURES UNIT 1

Preposition Logic

© Bharati Vidyapeeth's Institute of Computer Applications and Management , New Delhi-63, by Parul arora

Propositions

A proposition or statement is a declarative sentence which is either true or false, but not both.

For example:-

1. Paris is in France
2. 1+1=2
3. 2+2=3
4. 9<6
5. X=2 is a solution of $x^2 = 4$

Two letters T and F which are use to represent true and false statement are called propositional constant.

Compound Propositions

- Proposition which are composed by more then one propositions are called compound proposition.
- A proposition is said to be primitive if it can not be broken down in to simpler propositions.
- Examples:-
  - Roses are reds and violets are blue.
  - John is intelligent or studies every night.
- By the use of connectives we can combined the propositions.

Compound Propositions

Symbol	Connective word	Name
¬	not	negation
∧	And	conjunction
∨	or	disjunction
⇒	If…..then	Implication or conditional
⇔	If and only if	Equivalence or biconditional

Conjunction

- Conjunction (and): The conjunction of propositions p and q is the compound proposition "p and q".
- We denote it by p q $\wedge$ q . It is true if p and q are both true and false otherwise.
- For instance the compound proposition "2+2=4 and Sunday is the first day of the week" is true, but "3+3=7 and the sun rises in the east " false.
- The truth table that defines conjunction is (next slide)

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- The disjunction of propositions p and q is the compound proposition "p or q".
- We denote it by p Vq. It is true if p is true or q is true or both.
- For instance the compound proposition "2+2=4 or Sunday is the first day of the week" is true, and "3+3=7 or the sun rises from the east " is also true, but "2+2=5 or delhi is capital of england " is false.
- The truth table that defines disjunction is

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

- The negation of a proposition p is “not p” . It is true if p false and vice versa.
- This differs from the previous operators in that it is a *unary* operator, acting on a single proposition rather than a pair (the others are *binary* operators).
- Double negation of any statement gives the original statement.
- When a compound statement is negated its logical connectives changes from and to or and from or to and.

Negation

p	~p
T	F
F	T

Conditional (implies, if-then)

- Statement like
  - if you read, then you will pass the exam
  - if 9+3=12 then 12-3=9
- In all the above statement two simple statements are connected with if and then these type of statements are called conditional statement or implication.
- In it the statement p is called hypothesis and q is called conclusion.

Conditional (implies, if-then)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement

- The biconditional of propositions p and q is the compound proposition "p if and only if q" or "p is necessary and sufficient for q".
- We denote it by $p \Leftrightarrow q$ it can also be written as $(p \Rightarrow q) \wedge (q \Rightarrow p)$

Biconditional Statement

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p

Truth Tables

- A truth table lists the truth values of a compound proposition for every combination of truth values of its component simple propositions.
- Truth tables are a basic tool of propositional logic.
- For example, suppose we want the truth table for

$$p \rightarrow (\neg q \vee (p \wedge r))$$

- Here is the truth table we get, building up one operator at a time

.

Truth Tables

p	q	r	~q	$p \wedge r$	$\neg q \vee (p \wedge r)$	$p \rightarrow (\neg q \vee (p \wedge r))$
T	T	T	F	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

p

Tautologies

Some compound propositions have a truth value of true regardless of the truth values of their component propositions. That is, their truth tables show them being true on every line. Such propositions are called *tautologies*

OR

Truth table of a tautology will contain only T entries in the last column.

Tautologies

p	~p	$p \vee$ □ p
T	F	T
F	T	T

Contradiction

Truth table of a contradiction will contain only F entries in the last column

p

Algebra Of Propositions

Idempotent laws
p ∧ p ≡ p
p ∨ p ≡ p

Associative law
(p ∨ q) ∨ r ≡ p ∨ (q ∨ r)
(p ∧q) ∧ r ≡ p ∧ (q ∧ r)

Commutative law
p ∨ q ≡ q ∨ p
p ∧ q ≡ q ∧ p

Distributive law
p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)
p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r)

Identity Law
p ∨ T ≡ T   b) p ∧ T ≡ p
c) p ∨ F ≡ P   d) p ∧ F ≡ F

Complement law
p ∨¬ p ≡ T b) p ∧ ¬ p ≡ F
c) ¬ T ≡ F   d) ¬ F ≡ T

Involution law
¬ ¬ p ≡ p

DeMorgan's law
¬(p ∨ q) ≡ ¬ p ∧ ¬ q
¬(p ∧ q) ≡ ¬ p ∨ ¬ q

Propositional Logic

Logical Equivalence

- Two expressions are logically equivalent if they have the same truth table.

.

- For example, the expressions $p \rightarrow q$ and $\neg p \lor q$ are logically equivalent as the following truth tables show.
- We use the symbol $\equiv$ to indicate logical equivalence

p	q	$p \rightarrow q$	$\neg p \lor q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

p

Propositional Function

Let A be a given set. A propositional function defined on A is an expression p(x) which has the property that p(a) is true or false for each a ∈ A . Means p(x) becomes a statement whenever any element a ∈ A is substituted for the variable x. the set A is called the domain of p(x) and the set Tp of all elements of A for which p(a) is true is called the truth set of p(x)

Universal Quantifier

Let p(x) be a propositional function defined on a set A . Consider the expression

$(\forall x \in A)\ p(x)$ or $\forall x p(x)$

Which reads "For every x in A, p(x) is true statement"

This symbol "for all" or "for every" is called universal quantifier.

Existential Quantifier

Let p(x) be a propositional function defined on a set A then the expression

$(\exists x \in A)\ p(x)$ or $\exists x, p(x)$

Which reads "there exists an x in a such that p(x) is a true statement ". And this symbol ∃ is called existential quantifier.

Normal Forms

Disjunctive Normal Form

Conjunctive Normal Form

Set Theory

Learning Objective

- To discuss Uses Of Computer Networks
- To discuss features of Network Hardware
- To study charecterstics of LAN, MAN and WAN
- Features of wireless systems
- To study network software
- To deduce Layers, Protocols and Services

Set

'A mathematical *set* is defined as an unordered collection of distinct elements which share some common property. That is, elements of a set can be listed in any order and elements occurring more than once are equivalent to occurring only once'

For a set A having an element x,

Is written by $x \in A$

The following are all used synonymously:

x is a member of A

x is contained in A

x is included in A

x is an element of the set A

If d is not an element of set A then it is denoted by

$d \notin A$

Specifying Sets

Two notations are used for specifying a set

1)Roster method (Tabular form) Enumeration:- in it a set is represented by listing all its elements with in braces {}.

ex. The set of vowels in English alphabet

S= {a, e, i, o, u}

2)Rule method(Set-builder form) Symbolically:- in it a set is represented by describing its elements in terms of one or more several characteristics properties which enable us to decide whether a given object is an element of the set under consideration or not .

Ex. The set of vowels in English alphabet

S={x:x is a vowel in English alphabet}

3)Venn diagram

NOTATION FOR SOME FREQUENTLY USED SET

N=the set of natural number: 1,2,3…..

Z^+= the set of positive integer: 1, 2, 3……

Z= the set of integers: ….-2, -1, 0, 1, 2…….

Q= the set of rational numbers …-1/2, -1/5, 0, 1/3, 1/4 …..

R= the set of real numbers

C= the set of complex numbers

∑=set of alphabets

I=the set of whole numbers

Cardinality Of A Set

If there are exactly n distinct elements in set S where n is a nonnegative integer ,then S is a finite set and n is the cardinality of a set S .

Ex. S={2, 4, 6, 8, 10, 12, 14}

n(S) = 7

Types Of Set

1) **EMPTY/NULL SET:-**The set with no element is called empty or null or void set.it is denoted by ∅.

in Roster method ∅,is denoted by {}.

2) **UNIVERSAL SET:-**the member of all set always belong to some large set that is called universal set

it is denoted by U.

3) **SINGLETON SET :-**A set consist of single element is called singleton set.

ex. {∅} is a set whose only element is null therefore it is a singleton set.

4) **FINITE SET:-**A set is finite if it consist of finite number of different element ,means which should be countable.

ex. Set of week days

Types Of Set

5) **INFINITE SET:-**A set is infinite if it consist of infinite number of different element ,means which should not be countable.

6) **EQUIVALENT SET**:-Two finite set A and b are equivalent if their cardinal number are same.

means n(A) = n(B)

7) **EQUAL SET:-** Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A. ex. If A={2,3,4,5} and B={5,4,3,2}

then A and B are equal

IMP.-All equal set are equivalent but Vice-Versa is not true.

8) Overlapping and disjoint Sets

Subset

If every element in a set A is also a member of set B then A is called subset of B.

A is a SUBSET of B if $x \in A \Rightarrow x \in B$

Subset is denoted by $A \subseteq B$

If at least one element of A does not belong to B then

$A \nsubseteq B$

Ex. the set of all odd positive integer less then 10 is a subset of the set of all positive integer less then 10 .

Ex. $N \subseteq Z \subseteq Q \subseteq R$

Imp point:-

1) The total no of subset for any set is 2^n where n is the no of elements in that set.
2) ∅ Is a subset for each and every set

Proper Subset

Let A and B be two sets .If B ⊆ A and B # A

Means at least one element is there in set A, which is not in set B

then B is said to be proper subset of A.

It is denoted by B ⊂ A

suppose A={4, 5, 6} and B={5, 6}

then B is proper subset of A because all the elements of B are in A but one element 4 of A is not in B.

POWER SET

Set of Sets: If all elements of a set are sets

Mixed Set : If some elements are primitive elements and some are sets

The power set of set S is set of all possible subsets of the set S . It is denoted by P(S)

Ex: S={0,1,2}

P(S)={Ø, {0} , {1} , {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }

Empty set and the set itself are always member of the power set.

IMP:-

1) If a set has n element then its power set has 2^n elements

Q

Multi set

- Multi Set : A set in which an element may appear more than once
- Multiplicity : Frequency of appearance of an element x in a set is called multiplicity of x
- A set is special case of multiset in which muliplicity of every element is one

COUNTABLE INFINITE and UNCOUNTABLE INFINITE SET

Given two sets P and Q, we say that there is a one to one correspondence between the elements in P and elements in Q if it is possible to pair off the elements in P and Q such that every element in P is paired off with a distinct element in Q.

A set whose elements can be put in one one correspondence with the elements of the set N, then it is called **countable infinite or denumerable set**.

Ex: Set of all nonnegative even integer .{0,2,4,6,8……}

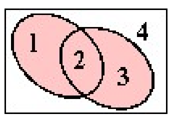
Set of all nonnegative multiple of 7 {0,7,14,21…..}

A set which can not be put in one to one correspondence with the element of set N, are called **uncountable infinite set or non denumerable set.**

Ex: Set of all real number between 0 and 1 .

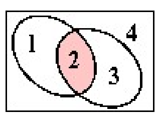
Pictorial Representation Of Set

UNION OF SETS suppose A and B are two set .The set which contain every element contained in A or B or Both A and B is called Union or join of A and B . Symbol U is use to represent Union .



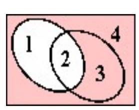
A U B= {x:x ∈A or x ∈ B}

INTERSECTION OF SETS The set containing all the elements which are contained in A as well as in B. Symbol ∩ is use to show intersection.



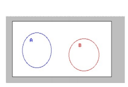
A ∩ B={x:x ∈ A and x ∈ B }

COMPLEMENT OF A SET The complement of a given set A is defined as the set consisting of those elements of the universal set which are not contained in the given set A it is denoted by symbol A'.



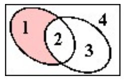
A' = {x:x ∈ U, x ∉ A}

DISJOINT SETS Two sets A and B are said to be disjoint sets if they have no element in common means,there intersection is null set.



A ∩ B = Ø

DIFFERENCE OF TWO SETS Diff of two sets written as A – B is the set of all those elements of A which do not belong to B.



$A - B = \{x: x \in A \text{ and } x \notin B\}$

SYMMETRIC DIFFERENCE OF TWO SETS

The symmetric difference of sets A and B contain those elements that are in P or in Q but not in both.i.e.

$(P \cup Q) - (P \cap Q)$

Also denoted by $P \Delta Q$ or $P \oplus Q$.

 U1.

PARTITIONS OF SET

Let S be a nonempty set . A partition of S is a subdivision of S in to **non-overlapping nonempty** subsets.

1) $S=A1\cup A2 \cup A3 \cup \ldots\ldots\ldots\ldots \cup An$
2)Non overlapping means $A1 \cap A2 = \emptyset$

Q

 U1.

LAWS OF ALGEBRA OF SETS

1) Idempotent Laws: a) $A \cup A=A$ b) $A \cap A=A$
2) Identity Laws a) $A \cup \emptyset = A$ b) $A \cap U = A$
3) Commutative Laws a) $A \cup B= B \cup A$ b) $A \cap B= B \cap A$
4) Associative Laws a) $(A \cup B) \cup C= A \cup (B \cup C)$
   b) $(A \cap B) \cap C= A \cap(B \cap C)$
5) Distributive Laws: a) $A \cup (B \cap C)= (A \cup B) \cap (A \cup C)$
   b) $A \cap (B \cup C)= (A \cap B) \cup (A \cap C)$
6) De Morgan's Law a) $(A \cup B)'=A' \cap B'$
   b) $(A \cap B)'=A' \cup B'$

 U1.

Cartesian Product

If A and B are two sets, then set of all distinct ordered pairs whose first coordinate is an element of A and whose second coordinate is an element of B is called the Cartesian product and it is denoted by A X B.

symbolically A X B={(a,b): a ∈ A and b ∈ B}

Characteristics of Cartesian product

A X B # B X A i.e. Cartesian product is not commutative.

If the set A and B have m and n elements respectively, then the set A X B has mn elements

A X B= B X A ➔ A=B

Ordered Pairs

An ordered pair of object is a pair of object arranged in some order. If x and y be any two elements ,then (x,y) is called their ordered pair. The element x is said to be the first member or the first coordinate of(x,y) and the element of y is called the second member of the ordered pair (x,y).

(a,b)=(c,d) means a=c and b=d.

Principal Of Inclusion And Exclusion

- If A and B be 2 sets then
  $|A \cup B| = |A| + |B| - |A \cap B|$
- If A and B be 3 sets then
  $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Mathematical Induction

Mathematical induction is a method which is used to prove the truthfulness of result which are obtained by any other method .it is not a tool for finding any formula.

Mathematical Induction

Steps involve in mathematical induction

First we prove that the result is true for n=1.

Then we assume that the result is true for n=r.

Finally we prove that the result is true for n=r+1.

Then we conclude by the principle of mathematical induction that the statement is true for all $n \in N$.

Counting Principle

a) $n(A \cup B)=n(A)+n(B)-n(A \cap B)$

this is also called inclusion-exclusion principle

b) $n(A \cup B)=n(A)+n(B)$ for disjoint sets

c) $n(A-B)=n(A)-n(A \cap B)$

d) $n(A \cup B \cup C)=n(A)+n(B)+n(C)-n(A \cap B)-n(B \cap C)-n(A \cap C)+n(A \cap B \cap C)$

Multiset

Multiset is a collection of elements that are not necessarily distinct.

MULTIPLICITY:- multiplicity of an element in multiset is defined by the number of times the element appears in the set.

Ex: {a,a,a,c,c,d,d} multiplicity of a = 3
multiplicity of c = 2

CARDINALITY:- cardinality is taken by just assuming it as a simple set.

Ex: {a,a,a,c,c,d,d} cardinality is = 3

Operations On Multiset

UNION:- P ∪ Q is equal to maximum of multiplicities of the element in P and in Q.
Ex:-P={a,a,a,c,d,d} Q={a,a,b,c,c}
P ∪ Q={a,a,a,b,c,c,d,d}

INTERSECTION:- P ∩ Q is equal to minimum of multiplicities of the elements in P and in Q.
Ex: P ∩ Q = {a,a,c}

DIFFERENCE:- multiplicity of an element in P-Q is equal to multiplicity of the element in P minus the multiplicity of the element in Q if diff is positive and is equal to 0 if diff is 0 or negative. .
Ex: P-Q= { a,d,d}

SUM:- multiplicity of the element in P+Q is equal to sum of the multiplicities of the element in P and in Q
Ex:- P+Q ={ a,a,a,a,a,b,c,c,c,d,d}

Relations

Relation

Let A and B be sets. A binary relation or simply relation from A to B is a subset of A X B

Suppose R is a relation from A to B. Then R is a set of ordered pairs where each first element comes from A and each second element comes from B.That is ,for each pair $a \in A$ and $b \in B$ exactly one of the following is true

$(a,b) \in R$: a is R-related to b

$(a,b) \notin R$: a is not R-related to b.

Domain And Range Of A Relation

Let R be a Relation from A to B

DOMAIN:- Set of all first co-ordinates of the members of the relation set R.

Domain **R={x : (x , y)** $\in$ R}

RANGE:- Set of all second co-ordinates of the members of relation set R.

Range R={y : (x, y) $\in$ R}

Total Number Of Relations

TOTAL NO OF RELATION

Let A and B be two non-empty sets consisting m and n elements respectively.

Then A X B has mn ordered pair.

Total no of subset of AX B is 2^mn

Since each subset of A X B defines a relation so total no of relation from A to B is 2^mn

Inverse Relation

Let R be any relation from set A to set B .The inverse of R, denoted by R^{-1}, is the relation from B to A which consist of those ordered pairs which when reversed belong to R : that is

$R^{-1} = \{(b,a): (a,b) \in R\}$

DOMAIN (R^{-1}) = RANGE (R)

RANGE (R^{-1}) = DOMAIN (R)

Is R is any relation then $(R^{-1})^{-1}$ is equal to R

Composition Of Relation

Let A ,B and C be sets and Let R be a relation from A to B and S be a relation from B to C. That is R is a subset of AXB and S is a subset of BXC. Then R and S give rise to a relation from A to C denoted by RoS

and defined by

RoS $=\{(a,c)$: there exists $b \in B$ for which $(a,b) \in R$ and $(b,c) \in S\}$

This relation is called composition of R and S. Sometime denoted by RS also.

suppose R is a relation from a set A to itself then R • R the composition of R with itself is defined by R^2 .

Types Of Relation

1)**<u>Reflexive Relation</u>:-**A relation R on a set A is said to be reflexive if every element of A is related to itself .

means R is reflexive $(a,a) \in R$ for all $a \in A$

A relation R on a set A is not reflexive if there exist an element $a \in A$ such that $(a,a) \notin R$

Ex. The relation R on N defined by $(x,y) \in R \Leftrightarrow x \geq y$ is a reflexive relation on N ,because every natural number is greater than or equal to itself.

4) Antisymmetric Relation:- Let A be any set ,A relation r on set is an antisymmetric relation iff

(a,b) $\in$ R and (b,a) $\in$ R then a=b for all a,b $\in$ A

Ex. Let r be a relation on the set N of natural number defined by x R y such that x divides y for all x,y $\in$ N

This relation is an antisymmetric relation on N because for any two number a,b $\in$ N

a / b and b / a $\Rightarrow$ a=b i.e. a R b and b R a $\Rightarrow$ a=b

5) Equivalence Relation:- A relation R on set A is said to be an equivalence relation on A iff

a)it is reflexive , symmetric and transitive

Properties of Equivalence relation

5) Equivalence Relation:- A relation R on set A is said to be an equivalence relation on A iff

a) It is reflexive, symmetric and transitive

Properties of Equivalence relation

a) if R is an Equivalence relation on a set A, then R^{-1} is also an equivalence relation on A

Closure Property

Reflexive closure:- R U Δ_A is reflexive closure of R.

reflexive (R) is obtained by simply adding to R those elements (a,a) in the set which do not already belong to R .

Symmetric closure:- R U R^{-1} is symmetric closure of R.

obtained by adding to R all pairs (b,a) whenever (a,b) belongs to R.

Transitive closure:- R^*

$$R^* = R \cup R^2 \cup \ldots \cup R^n$$

Partitions Of A Set

Let X be a non empty set. A set P={A,B,C} of nonempty subset of X will be called a partition of X iff

1) A ∪ B ∪ C……= X, i.e. the set X is the union of the sets in P and
2) The intersection of every pair of distinct subsets of X ∈ P

is the null set i.e.

if A and B ∈ P then either A=B or $A \cap B = \emptyset$

Partial Order Relation

A relation $\leq$ on a set A is called a partial order relation iff it is

Reflexive, antisymmetric and transitive and in this case the set is called partially ordered set and is denoted by symbol

$(A, \leq)$.

COMPATIBLE RELATION

A Relation on a set that is reflexive and symmetric is called compatible relation.

Imp:-Every equivalence relation is an compatible relation but every compatible relation need not be equivalence relation.

Ex. Let a,b,c be elements of A and $(a,b) \in R$ and $(b,c) \in R$.

suppose a is a friend of b and b is a friend of c then it may or may not be possible that a is a friend of c or not.

So relation can or can not be transitive.

Congruence Modulo M

Let m be an arbitrary and fixed integer.Two integers a and b are said to be congruence modulo m if a-b is divisible by m

$$a \equiv b \pmod{m}$$

Imp:-relation congruence modulo m on the set Z of all integers is an equivalence relation.

Function

Function

In layman terms

A function is nothing but a relationship between two sets.It maps element of one set to another set on the basis of some logical relation.

But in terms of mathematics

A binary relation R from A to B is said to be a function, if for every element a in A, there is a unique element b in B so that (a, b) is in R.

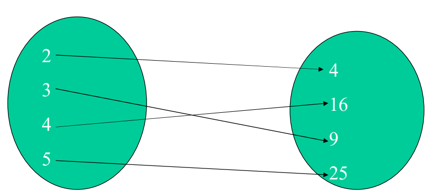
We use the notation R(a)=b, where b is called the image of a.

Diagrammatic Representation Of Function

A={2,3,4,5} and B={4,9,16,25}

f : A--->B

f(x) = x^2



Rules For Function

1) There may be some element of set B which are not associated to any element of Set A but each element of set A must be associated to only one element of set B

   for each $a \in A$, $(a,b) \in f$, for some $b \in B$

2) Two or more element of set A may be associated to same element of set B but association of one element of A to more then one element in B is not possible

   if $(a,b) \in f$ and $(a,b') \in f$ then $b=b'$

Domain Co-domain And Range

Suppose f be a function from A to B

Domain:- A is called the domain of the function f.

CO-Domain:- B is called the co-domain of function.

Range:- it consist of all those elements in B which appear as image of at least one element in A.

Type Of Functions

One-to-one function (injection):- if different elements of A have different images in B. For f:-A→B f(a) # f(b) for all $a,b \in A$

Many-to-one function:- if two or more elements of Set A has same images in B.For f:-A→B there exists $x,y \in A$ such that x # y but f(x)=f(y).

Onto function (Surjection):-if every element of B is the image of some element of A. In this case range of f is co-domain of f.

Into function:-if there exists an element in B which is not the image of any element of A.

Bijection (one-to-one onto function):-if function is one-to-one as well as onto then it is bijection.

One-to-one into mapping:- Function which is one-to-one as well as into is one-to-one into Function.

Many-to-one into mapping:- Function which is many-to-one as well as into is called many-to-one into mapping.

Many-to-one onto mapping:- Function which is many-to-one as well as onto is called many-to-one onto mapping

Ex.-Let A be the set of students sitting on chairs in a class room and let B be the set of chairs in a class room.Let f be the correspondence which associates to each student the chair on which he sits. Since every student has some chair to sit on(of course two or more than two students might sit on one chair) and no student can sit on two or more then two chairs,

Therefore f:A→B

1) If every student gets a separate chair and no chair is left vacant,then this is a case of **one-one onto mapping.**
2) If every student gets a separate chair and still some chair lie vacant it is a **one-one into mapping.**
3) If every student does not sit on separate chair and no chair is left vacant ,then this is a case of **many one onto** mapping.
4) If every student does not sit on separate chair and some chair are left vacant then this is a case of **many one into** mapping

Inclusion Map

The function f:X→Y defined by f(x)=x for each

x ∈ X is called inclusion map.

Ex- Let X={-3,-2,-10,1,2,3} and

Y={…-4,-3,-2,-1,0,1,2,3,4--- }

And f(-3)=-3 , f(-2)=2 , f(-1)=-1 , f(0)=0 , f(1)=1, f(2)=2

f(3)=3

Then it is an inclusion map from X to Y because X ⊂ Y and f(x)=x for each x ∈ X

Inverse Image Of An Element

Let f : X→Y then inverse image of an element b ∈ Y under f is denoted by f^{-1} (b) to be read as f-image b and

f^{-1} (b)={x:x ∈ X and f(x)=b}

INVERSE IMAGE OF A SUBSET

Let f : X→Y and B be a subset of Y means B Y then the inverse of B under f is given by

f^{-1} (b)= { x:x ∈ X and f(x) ∈ b}

Inverse Mapping Or Function

Let *f* be a function whose domain is the set *X*, and whose range is the set *Y*. Then, if it exists, the **inverse** of *f* is the function f^{-1} with domain *Y* and range *X*, defined by the following rule:

If f(x)=y then f^{-1} (y)=x

A function is **invertible** if it's inverse relation is also a function.

Composite Function

Let f : X→Y and g : Y→Z then the composite of the function f and g denoted by gof is mapping gof :X→Z s.t.

(gof)(x)=g[f(x)]

Ex:-Let f:R→R and f(x)=sin x

and g : R→R and g(x)=x^2

Then composite function

(gof)(x) = g[f(x)] = g[sin x] = (sin x)^2

(fog)(x) = f[g(x)] = f[x^2] =sin x^2

Imp:- product (gof) is defined only when

range (f) $\subset$ domain(g)

Ex:-if the mapping f and g are given by

f={(1,2),(3,5),(4,1)}

g={(2,3),(5,1),(1,3)}

Then find out the pairs in mappings (fog) and(gof)

Recursively Defined Functions

If the function definition refers to itself then it is called recursively defined function

It must have two properties

1) There must be certain argument,called base value for which the function does not refers to itself.
2) Each time the function does refer to itself,the argument of the function must be closer to a base value.

Example Of Recursively Defined Function

1) **<u>Factorial function</u>:-** the product of positive integer from 1 to n is called n factorial and denoted by n!.

n!=1.2.3…(n-2)(n-1)n

we can define factorial function recursively like this

a) If n=0,then n!=1.
b) If n>0,then n!=n.(n-1)!

From above definition we can say that

a) The value of n! is given when n=0. Thus 0 is a base value.
b) The value of n! for any n is defined in terms of a smaller value of n which is closer to the base value 0.

Fibonacci Sequence:- The fibonacci sequence ,usually denoted by fn is as follows

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55…..

i.e. F0=0, F1=1 and each succeeding term is the sum of two preceding terms .

Definition:-

If n=0 or n=1 then Fn=n.

If n>1 then Fn=Fn-1 + Fn-2.

In this recursive function

The base value are 0 and 1.

The value of Fn is defined in terms of smaller values of n which are closer to the base value.

Ackermann function:- This is a function with two arguments ,each of which can be assigned any non-negative integer.it is defined as follows

Definition:-

If m=0 then

A(m, n)=n+1

b) If m # 0 and n=0 then

A(m,n)=A(m-1,1)

c) If m # 0 and n # 0 then

A(m,n)=A(m-1),A(m, n-1)

BHARATI VIDYAPEETH PUNE

Proof

Direct Proof

The implication p---- > q can be proved by showing that if P is true then q must also be true means combination like p true and q false never occurs.

Ex:- if n is an odd integer then n^2 is an odd integer

Assume that hypothesis of a question is true means n is an odd integer .

N=2K+1

$N^2 = [2k+1]^2$

$= 4k^2 + 1 + 4k$

$= 2(2k^2+2k) + 1$ which is an odd number

Proof by Contrapositive /indirect Proof

For example, the assertion "If it is my car, then it is red" is equivalent to "If that car is not red, then it is not mine". So, to prove "If P, Then Q" by the method of contrapositive means to prove "If Not Q, Then Not P". **Steps for contrapositive method** :-First assume q is false. Then prove on the basis of the assumption and other available information that p is false

Ex:- if 3n+2 is odd , then n is odd

Assume that the conclusion (q) of this implication is false

Means n is even (n=2k) for some integer k.

Put the value of n in equation

3(2k) + 2= 6k+2 = 2(3k+1) so, 3n+2 is even because it is a multiple of 2 and therefore not odd .

So we can say in any case if 3n+2 is odd then n must be odd.

Example:- Prove that if n is an integer and n^3+5 is odd, then n is even

Ans:- Assume that the conclusion (q) of this implication is false

means assume n is odd

then n=2k+1

means $n^3+5 = (2k+1)^3 +5$

$n^3+5 = = 2(4k^3+6k^2+3k+3)$

Which is an even number

So, we can say that it is necessary if n3+5 is odd then n must b even

Proof By Contradiction

For p ---- > q type of statement

Steps:-

First assume p ∧ (~ q) is true

Then find some conclusion that is patently false or violates some other facts already established .

Then the contradiction find out in step (b) leads us to conclude that p ∧ (~ q) is false

For a single statement

Given a statement p, assume it is false

Assume ¬p

Prove that ¬p cannot occur

A contradiction exists

Then we can say that p was true . That is basically a method of proof by contradiction

EXAMPLE:

Prove that the sum of an even integer and a non-even integer non-even.

Assume that [a is even and b is non-even], ∧ [a+b is even].

So for some integers m,n,

a=2m and a+b=2n.

Since b=(a+b)-a, b=2n-2m=2(n-m).

We conclude that b is even. This leads to a contradiction, since we assumed that b is non-even.

Difference Between Contrapositive And Contradiction Method

Let's examine how the two methods work when trying to prove "If P, Then Q".

Method of Contradiction: Assume P and Not Q and prove some sort of contradiction.

Method of Contrapositive: Assume Not Q and prove Not P.

The method of Contrapositive has the advantage that your goal is clear: Prove Not P. In the method of Contradiction, your goal is to prove a contradiction, but it is not always clear what the contradiction is going to be at the start.

Example:- Prove that if n is an integer and n^3+5 is odd, then n is even

Rephrased: If n^3+5 is odd, then n is even

Thus, p is "n^3+5" is odd, q is "n is even"

Assume $p \wedge \neg q$

Assume that n^3+5 is odd, and n is odd

Since n is odd:

n=2k+1 for some integer k (definition of odd numbers)

$n^3+5 = (2k+1)^3+5 = 8k^3+12k^2+6k+6 = 2(4k^3+6k^2+3k+3)$

As n3+5 = $2(4k^3+6k^2+3k+3)$ is 2 times an integer, n must be even.

It shows contradiction .

Basic Counting Technique

Basic Counting Technique

Sum Rule Principle Suppose an event E can occur in n1 ways and a second event F can occur in n2 ways and if both event can not occur simultaneously . Then E or F can occur in n1+n2 ways.

GENERAL FORMAT suppose an event E1 can occur in n1 ways , a second event E2 can occur in n2 ways , a third event E3 can occur in n3 ways ….. And suppose no two of the event can occur at the same time. Then one of the event can occur in n1+n2+n3…. Ways.

Ex:- suppose there are 8 male professor and 5 female professor teaching a calculus class. A student can choose a calculus professor in 8+5 =13 ways.

Ex:- Suppose E is the event of choosing a prime no les then 10, and suppose F is the event of choosing an even number less then 10 . By how many ways E or F can occur.

Product Rule Principle

If an event E can occur in m ways and, independent of this event there is a second event F which can occur in n ways . Then combination of E and F can occur in m. n ways. (the product rule applies when a procedure is made up of separate task).

GENERAL FORMAT Suppose an Event E1 can occur in n1 ways and following E1 event E2 can occur in n2 ways and following E2 a third event E3 can occur in n3 ways and so on. Then all the event can occur in n1.n2.n3….ways

OR

Suppose that a procedure can be broken in to a sequence of two tasks . If there are n1 ways to do the first task and n2 ways to do the second task after the first task has been done then there are n1n2 ways to do the procedure.

Ex Suppose a license plate contain two letters followed by three digit with the first digit not zero . How many diff license plates can be printed.

 U1.

Suppose there are 5 different optional papers to select in third semester and 4 different optional papers to select in 4th semester by the MCA students.

By product rule:- there will be 5*4 choices for students who want to select one paper in third semester and one in 4th semester.

By sum rule:- student will have 5+4 choices to select only one paper

Q A football stadium has five gates on the eastern boundary and 4 gates on the western boudary.

In how many ways can a person enter through an east gate and leave by a western gate

In how many different ways in all can a person enter and get out through different gates.

 U1.

Q how many 7 digit telephone no are possible ,if

Only odd digit may be used

The number must be a multiple of 100

The first three digits are 481

 U1.

Permutation

Any arrangement of a set of n objects in a given order is called a permutation of the object (taken all at a time)

The number of permutation of n object taken r at a time is denoted by

P (n, r) = !n / !(n - r)

Imp:-1) there are !n permutation of n object.

2) number of permutation of n objects of which n1 are alike , n2 are alike ……..nr are alike . Then the general formula is

P(n1,n2,n3……..nr) = !n / (!n1.!n2.!n3….!nr)

3) n objects can be arranged in a circle in !(n-1) ways.

Q find out how many 5-digits number greater than 30,000 can be formed from the digit 1,2,3,4,5.

Q how many permutation can be made with the letters of the word CONSTITUTION and

In how many ways vowels occur togather

In how many ways consonant and vowels occur alternatively

How many of these will have the letter N both at the beginning and at the end.

Q in how many ways can the letters of the world ALLAHABAD be arranged? How many of these permutations in which

Two Ls come togather

Two Ls do not come togather

Combination

Combination of n object taken r at a time .

C(n ,r) = !n /!r !(n - r)

Q1 in how many ways can a cricket team of 11 be chosen out of a batch of 14 players . How many of them will

Include a particular player

Exclude a particular player

Q 2 There are 7 men and 3 women . Find the number of ways in which a committee of 6 persons can be formed if the committee is to have

Exactly 4 men

Atleast 2 women

Q3 an examination paper consist of 12 questions is divided in two parts A and B. Part A contain 7 questions and Part B contain 5 questions . A candidate is require to answer 8 questions electing atleast 3 questions from each part.in how many ways can the candidate select the questions.

Q4 out of 7 consonants and 4 vowels how many words can be made each containing 3 consonant and 2 vowels.

Q5 out of 5 males and 6 females , a committee of 5 is to be formed . Find the number of ways in which it can be formed so that among the person chosen in the committee there are :

Exactly 3 males and 2 females

Atleast 2 male and 1 female

Pigeonhole Principle

Pigeonhole principle also known as Drichlet drawer principle states that if there are more pigeons then pigeonholes ,then there must be atleast one pigeonhole with atleast two pigeons in it.

means

If n pigeonholes are occupied by n+1 or more pigeons ,then at least one pigeonhole is occupied by more then one pigeon.

Ex suppose a department contain 13 professor . Then two of the professor are born in same month.

GENERALIZED PIGEONHOLE PRINCIPLE :- if n pigeonholes are occupied by kn+1 or more pigeons , where k is a positive integer ,then at least one pigeonhole is occupied by k+1 or more pigeons.

Q. Find the minimum no of students in a class to be sure that three of them are born in the same month.